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Engineering Notes

Wrinkling Analysis Method Based on Singular Displacement Component Modification for Membrane Structure

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Nomenclature

K_T = tangential stiffness matrix
$K_{T\mathrm{cr}}$ = critical tangential stiffness matrix
K_T^* = modified critical tangential stiffness matrix
u = true solution of the original equation
u^* = modified displacement solution
u_ϕ = wrinkling mode
Δu = tangential displacement increment
α = imperfection scaling
β = modified parameter
$o()$ = higher-order infinitesimal
ϕ = first-order characteristic vector
ϕ_{cr} = critical wrinkling mode

I. Introduction

FLEXIBLE membrane structures have received wide attention due to their low package volume and the ability of constructing the ultralightweight aerospace structures, including the James Webb space telescope sunshield, solar sail, and inflatable reflector [1]. The wrinkles, as the unique phenomena, may significantly affect the performance and the reliability of gossamer space structures, and thus have received the widest research interests.

In the literatures, there are mainly three types of wrinkling models: membrane model, thin plate or thin shell model, and explicit time integration method. The membrane model was attributed to Wagner's tension field theory (TFT) [2]. The TFT was based on the assumption that wrinkles were oriented in the local major principal stress direction, and the minor principal stress in wrinkling region, which was perpendicular to the load path, was zero. Later, this theory was extended into two major groups: the deformation tensor modification [3–5] and the stiffness/compliance modification [6–8]. The region and direction were major results from membrane models while it was impossible to obtain the detailed wrinkling characteristics, such as amplitude and wavelength. However, the TFT was also widely used to predict the wrinkling stress and region due to its high computational efficiency and low mesh dependence until now [9,10]. To enable computational modeling of detailed wrinkling deformations, both membrane and bending stiffness must be considered based on nonlinear postbuckling analysis. Several recent computational studies had employed nonlinear shell models [11–15]. Two major drawbacks of this model were the sensitive mesh dependency and the difficult convergence. The complete survey of the membrane wrinkling literature was given in the book by Jenkins [16] and the paper by Wang et al. [17]. When the nonlinear shell model was used, a singular tangent stiffness matrix was encountered, which may result in poor convergence. Quasi-static modeling [18] with explicit time integration was an alternative approach, which did not require the inverse of a singular matrix. The explicit time integration method was also used to obtain the dynamic wrinkling behaviors in inflatable space structures [19].

In this paper, the newly developed modified displacement component (MDC) method is presented to compute the wrinkles as the core part of our previous paper [20]. This method focuses on the elimination of the singularity of the first-order characteristic vector in the characteristic vector space. A positive parameter multiplied by the first-order characteristic vector is introduced into the stiffness matrix to convert the elimination of the singularity of the tangential stiffness matrix into eliminating the singularity of the displacement component. The accurate introduction and the timely removing of the critical wrinkling mode are two key steps that are different from the prior literatures. In our simulation, we use a direct-perturbed force (DP) technology to accurately consider these two key steps. Several effective strategies are then used to advance the convergence. The wrinkling photogrammetry test results [21] are used to verify the validity of such methods in the end.

II. Modified Displacement Component Method

At the critical wrinkling point, the characteristic equation of nonlinear buckling analysis meets

$$K_{T\mathrm{cr}}\phi_{\mathrm{cr}} = 0$$

where $K_{T\mathrm{cr}}$ is the critical tangential stiffness matrix and ϕ_{cr} is the critical wrinkling mode. Such a relationship is the problem causing a singular stiffness matrix. We have to perform our analysis into the postwrinkling phase to obtain detailed wrinkling characteristics. Therefore, the elimination of the wrinkling singularity is our major task.

According to the characteristic vector space, the elimination of the singularity of stiffness matrix is mainly focused on the elimination of the singularity of the first-order characteristic vector. The column vector of the first-order characteristic vector is given by

$$\phi = (0, 0, \ldots, 1(ith), \ldots, 0)^T \tag{2}$$

Introducing a parameter $\beta > 0$ in the stiffness matrix at the wrinkling point, and the modified stiffness matrix is

$$K_T^* = K_T + K_\phi = K_T + \beta\phi\phi^T \tag{3}$$

The modified wrinkling control equation is further given by

$$K_T^* u = K_T u + \beta(\phi^T u)\phi = q + \beta(\phi^T u)\phi \tag{4}$$

where, u is the true solution of original equation. Then $(K_T^*)^{-1}$ is multiplied by all items in Eq. (4), and we have

$$u = (K_T^*)^{-1} q + \beta(\phi^T u)(K_T^*)^{-1} \qquad \phi = u_q + \beta(\phi^T u) u_\phi \tag{5}$$

where u_ϕ is the wrinkling mode, $K_T^* u_q = q$, and $K_T^* u_\phi = \phi$. Next, a

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vector ϕ^T is multiplied by all items in Eq. (5), and we get

$$\phi^T u = \phi^T u_q + \beta(\phi^T u)\phi^T u_\phi \tag{6}$$

Then we can obtain the following expression

$$\phi^T u = \frac{\phi^T u_q}{(1 - \beta\phi^T u_\phi)} \tag{7}$$

Thus, the original displacement solution u may be determined by substituting Eq. (7) in Eq. (5)

$$u = u_q + \frac{\beta\phi^T u_q}{1 - \beta\phi^T u_\phi} u_\phi \tag{8}$$

According to this result, the singularity mainly occurs at the right second item when $1 - \beta\phi^T u_\phi = 0$. Then we may obtain the nonsingular displacement solution by eliminating the singularity of the singular displacement component. The elimination of the singularity of displacement component should meet the following three equality relationships at the critical wrinkling point.

Firstly, for the critical wrinkling point, the wrinkling mode should be replaced by the first-order mode. So the first modified relationship is given by

$$u_\varphi = \varphi_1 \tag{9}$$

The first wrinkling mode is related to the occurrence of the initial wrinkle. It can be introduced in our model by using the direct perturbed method [20].

From the view of the physical meaning, there is no loading corresponding to the critical wrinkling mode component when wrinkling occurs. In other words, the force corresponding to the critical wrinkling mode should be timely eliminated after the wrinkle is induced. Otherwise, the numerical error from this component will be enlarged and directly lead to the computing divergence. Thus, we introduced the second modified relationship

$$u^T u_\varphi = u_q^T u_\varphi = 0 \tag{10}$$

This relationship also reveals that the elimination of the initial imperfections is based on the first wrinkling mode.

The third relationship can be obtained according to the existence of the real displacement solution. According to Eq. (8), the singularity will occur when $1 - \beta\phi^T u_\phi = 0$. Thus, we have to set the corresponding numerator is equal to zero at the wrinkling point to meet the existence of the displacement solution. Therefore, the third modified relationship can be expressed as

$$\varphi^T u_q = (K_T^* u_\varphi)^T u_q = u_\varphi^T q = 0 \tag{11}$$

Next, we will use these three relationships to eliminate the singularity of the displacement component. In Eq. (8), we assume $\frac{\beta\phi^T u_q}{1-\beta\phi^T u_\phi} = C$, C is a real constant. Thus, the Eq. (8) can be rewritten as

$$u - u_q = C u_\phi \tag{12}$$

Further given as

$$u^T - u_q^T = C u_\phi^T \tag{13}$$

The wrinkling mode u_φ is then multiplied by all items in Eq. (13), and we obtain

$$u^T u_\varphi - u_q^T u_\varphi = C u_\phi^T u_\varphi \tag{14}$$

Then we get

$$C = \frac{u^T u_\varphi - u_q^T u_\varphi}{u_\phi^T u_\varphi} \tag{15}$$

By introducing the second modified relationship, we further have

$$C = \frac{\beta\phi^T u_q}{1 - \beta\phi^T u_\phi} = \frac{-u_q^T u_\varphi}{u_\phi^T u_\varphi} \tag{16}$$

Here, we observe that $u_\phi^T u_\varphi > 0$. Thus, we obtain a nonsingular displacement component $C u_\phi$. Then, the nonsingular displacement solution u^*can be obtained by substituting Eq. (16) in the second item of Eq. (8)

$$u^* = u_q - \frac{u_q^T u_\phi}{u_\phi^T u_\phi} u_\phi \tag{17}$$

This process is the core part of the MDC method. Here the key problem is to accurately introduce the wrinkling mode, and timely remove it.

The postwrinkling load path is the bifurcation path, and the bifurcation path is transformed from the initial equilibrium path by introducing a suitable perturbation. Such perturbation is also related to the wrinkling mode. The bifurcation path can be expressed as the incremental displacement

$$\Delta u = u_T + \alpha u_\phi + o(\Delta u) \tag{18}$$

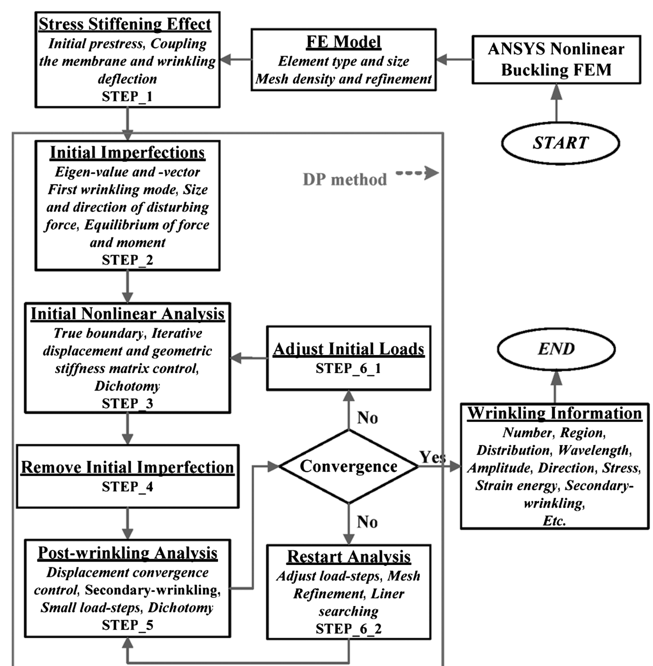
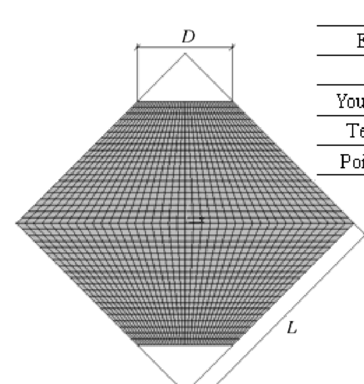


Fig. 1 Flowchart of wrinkling simulation.



Edge length, L	100mm
Thickness, h	0.05mm
Young's modulus, E	4.91GPa
Tension length, D	40mm
Poisson's ratio, ν	0.3

Fig. 2 Finite element model and parameters.

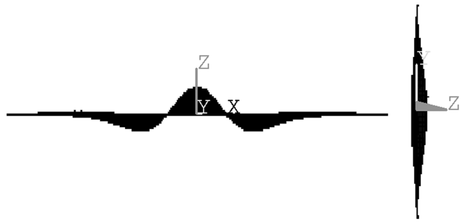


Fig. 3 The first-order wrinkling mode.

where Δu is the tangential displacement, αu_ϕ is the initial imperfection that is introduced to the wrinkling analysis to initiate the occurrence of the wrinkles, and $o(\Delta u)$ is a higher-order infinitesimal displacement that is related to the modification.

III. Wrinkling Simulation

The keys to the MDC method are the accurate introduction and the timely removal of the wrinkle mode. To accurately perform the wrinkling analysis, we introduce the cord idea of the MDC method by using a direct-perturbed force (DP) technology in our simulation. The basis of the DP technology is to apply some small out-of-plane forces on the membrane surface to induce the imperfections, and further to induce the wrinkle. After that, these imperfections are timely removed from the model by deleting those out-of-plane forces.

We should take care of two key problems. Firstly, the detailed imperfections should be based on the first wrinkling mode, which is obtained from an eigenvalue buckling analysis, including the imperfection amplitude, region, and direction. In other words, these imperfections generated by the out-of-plane forces have the quantitative characteristics, and they are not random distributions. Secondly, the equal numbers of positive and negative out-of-plane forces should be applied on the membrane surface so that the net out-of-plane forces remain equal to zero, which meets the force equilibrium condition and mainly initiates the analysis into the postwrinkling phase. The location of these out-of-plane forces are also based on the first wrinkle mode. DP method is thus restricted to wrinkling modes with an even number of wrinkle waves, so that they meet the moment equilibrium as well.

In addition, we also use several effective strategies to advance the convergence, including the stress stiffening effect, the displacement control technique, the Newton–Raphson iteration, and the dichotomy method. The flowchart of the wrinkling simulation is shown in Fig. 1 based on ANSYS nonlinear shell finite element analysis. A uniform mesh of 1600 thin shell elements was used to model the whole membrane structure (as shown in Fig. 2), in order to capture the fine wrinkle details in the membrane. The first wrinkling mode obtained from the eigenvalue buckling analysis is shown in Fig. 3. The wrinkling results under different tension loadings are shown in Fig. 4 based on our method.

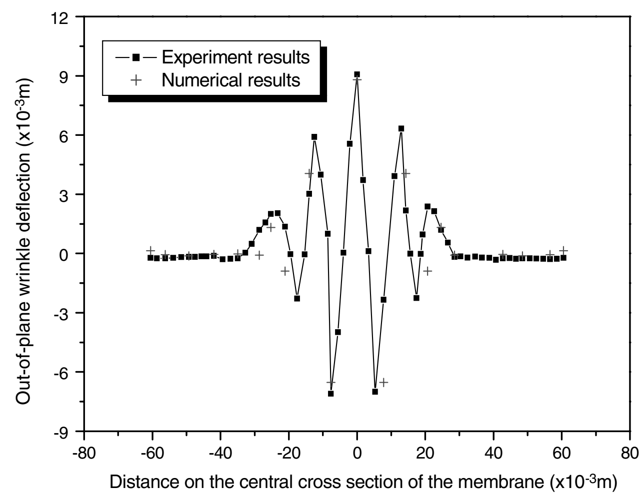
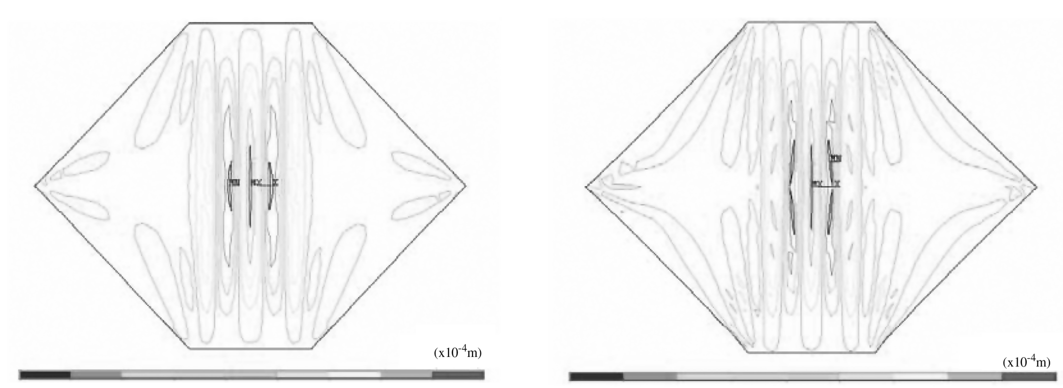
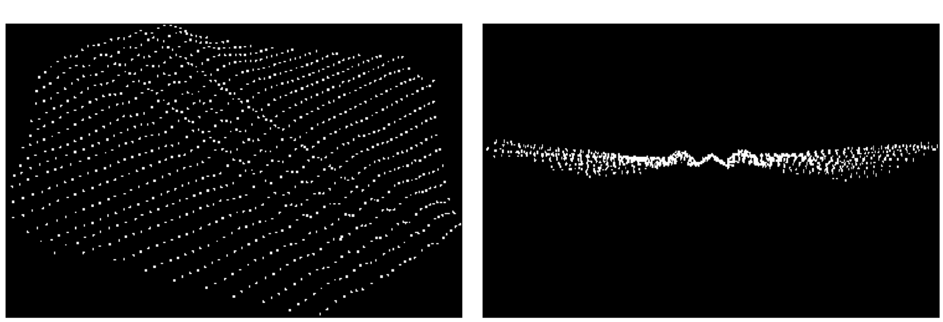


Fig. 6 Comparison of the wrinkling calculation and experiment (1 mm tension displacement load).



a) 0.5×10^{-3}m tension displacement load **b) 1×10^{-3}m tension displacement load**

Fig. 4 Wrinkling calculated results.



a) **b)**

Fig. 5 Wrinkling test results (1 mm tension load): a) solid view, and b) side view.

IV. Experimental Verification

The wrinkling experiment based on the photogrammetric test is performed to verify the wrinkling analysis, and the experimental results of the wrinkling configuration are plotted in Fig. 5 [21]. The out-of-plane wrinkling deflections on the central cross section from the experimental results and the wrinkling calculation are compared in Fig. 6. The detailed process of this experiment can be obtained in [21].

According to the comparison, the wrinkling prediction shows good agreement with the experimental results, and the regions and distributions of the wrinkles are also consistent. In addition, according to our test there are three wrinkling crests, the wrinkling wavelength is 1.3×10^{-2} m, and the wrinkling amplitude is 9.1×10^{-3} m. They are very close to the numerical results (wavelength 1.414×10^{-2} m and amplitude 8.77×10^{-3} m).

V. Conclusions

The core part of the new MDC method for wrinkling analysis is present in detail. Two key steps in MDC method associated with the DP technology are effectively used to simulate the membrane wrinkling. The numerical results are close to the experiment test data, which verify the accuracy of the MDC method for predicting the membrane wrinkling characteristics.

According to the results, the distribution of wrinkles is very uniform and concentrated. The wrinkle numbers, region, and amplitude are increasing with the increment of tension load, and the wrinkle wavelength inversely varies with the tension. Also, the wrinkles extend in transverse and longitudinal direction at the same time.

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